GL_n tensor product algebra and the Littlewood-Richardson rule

Soo Teck Lee
National University of Singapore

The algebra $\mathcal{P}(\mathrm{M}_{n,k+\ell})$ of polynomial functions on the space $\mathrm{M}_{n,k+\ell}$ of $n \times (k+\ell)$ complex matrices carries an action by $\mathrm{GL}_n \times \mathrm{GL}_k \times \mathrm{GL}_\ell$. Let U_n be the maximal unipotent subgroup of GL_n consisting of all upper triangular matrices with 1's on the diagonal. The groups U_k and U_ℓ are defined similarly. We consider the subalgebra $\mathcal{P}(\mathrm{M}_{n,k+\ell})^{U_n \times U_k \times U_\ell}$ of $\mathcal{P}(\mathrm{M}_{n,k+\ell})$ consisting of polynomials which are invariant under the action by $U_n \times U_k \times U_\ell$. This algebra can be used to study tensor products of GL_n representations, so it is called a *GL_n tensor product algebra*. In this talk, we will use this algebra to construct a proof of the the Littlewood-Richardson rule. This is joint work with Roger Howe.