Branching rules and branching algebras for the complex classical groups

Soo Teck Lee
National University of Singapore

Branching rules are descriptions of how irreducible representations of a group G decompose under restriction to a subgroup H. We are interested in the cases when G is a complex classical group. For a certain subgroup H of G, combinatorial descriptions of the branching multiplicities from G to H are known. For example, by the Littlewood-Richardson rule, certain skew tableaux are used to count the multiplicities in the tensor product of 2 irreducible representations of the general linear group $\mathrm{GL}_n = \mathrm{GL}_n(\mathbb{C})$.

The goal of this series of talks is to describe a new approach to study branching rules for the complex classical groups developed by Howe and his collaborators. Given a complex classical group G and a subgroup H of a certain type, one can construct using classical invariant theory an algebra $R_{(G,H)}$ whose structure encodes the branching rule from G to H. We call $R_{(G,H)}$ a *branching algebra* for (G, H). It turns out that $R_{(G,H)}$ has a very pleasant structure. In particular, $R_{(G,H)}$ has a distinguished vector space basis whose elements are indexed by the combinatorial objects used in counting the multiplicities.

The following topics will be discussed:

- Some basic definitions and results in representation theory and invariant theory.
- Standard monomial theory for GL_n.
- Branching algebras for $(\mathrm{GL}_n, \mathrm{GL}_m \times \mathrm{GL}_1^{n-m})$.
- Branching algebras for $(\mathrm{GL}_n, \mathrm{GL}_m \times \mathrm{GL}_{n-m})$.
- Other examples of branching algebras.